Uncertainty, Stochastics & Sensitivity Analysis

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Types of Sensitivity Analyses

- Variables involved
  - One-way
  - Multi-way
- Type of component being varied
  - Parameter sensitivity analysis: Parameter values
  - Structural sensitivity analysis: Examine effects of model *structure* on results
- Type of variation
  - Single alternative values
  - Monte Carlo analyses: Draws from probability distributions (many types of variations)
- Frequency of variation
  - Static (parameter retains value all through simulation)
  - Ongoing change: Stochastic process
    - Accomplished via Monte-Carlo analyses
    - Key for DES & ABM

Model Uncertainty

- Here, we are frequently examining the impact of changing
  - Our assumptions about “how the system works”
  - Our decision of how to abstract the system behaviour
- Structural sensitivity analyses
  - Vary structure of model & see impact on
    - Results
    - Tradeoffs between choices
  - Frequently recalibrate the model in this process
- Here, we are considering uncertainty about how the current state is mapped to the next state

Predictor-Corrector Methods: Dealing with an Incomplete Model

- Some approaches (e.g. Kalman filter, Particle Filter) are motivated by awareness that models are incomplete
- Such approaches try to adjust model state estimates on an ongoing basis,
  - Given uncertainty about model predictions
  - New observations
- Assumption here is that the error in the model is defined by some probability distribution

Static Uncertainty Sensitivity Analyses

- In variation, one can seek to investigate different
  - Assumptions
  - Policies
- Same relative or absolute uncertainty in different parameters may have hugely different effect on outcomes or decisions
- Help identify parameters/initial states that strongly affect
  - Key model results
  - Choice between policies
- We place more emphasis in parameter estimation & interventions into parameters exhibiting high sensitivity

Spider Diagram

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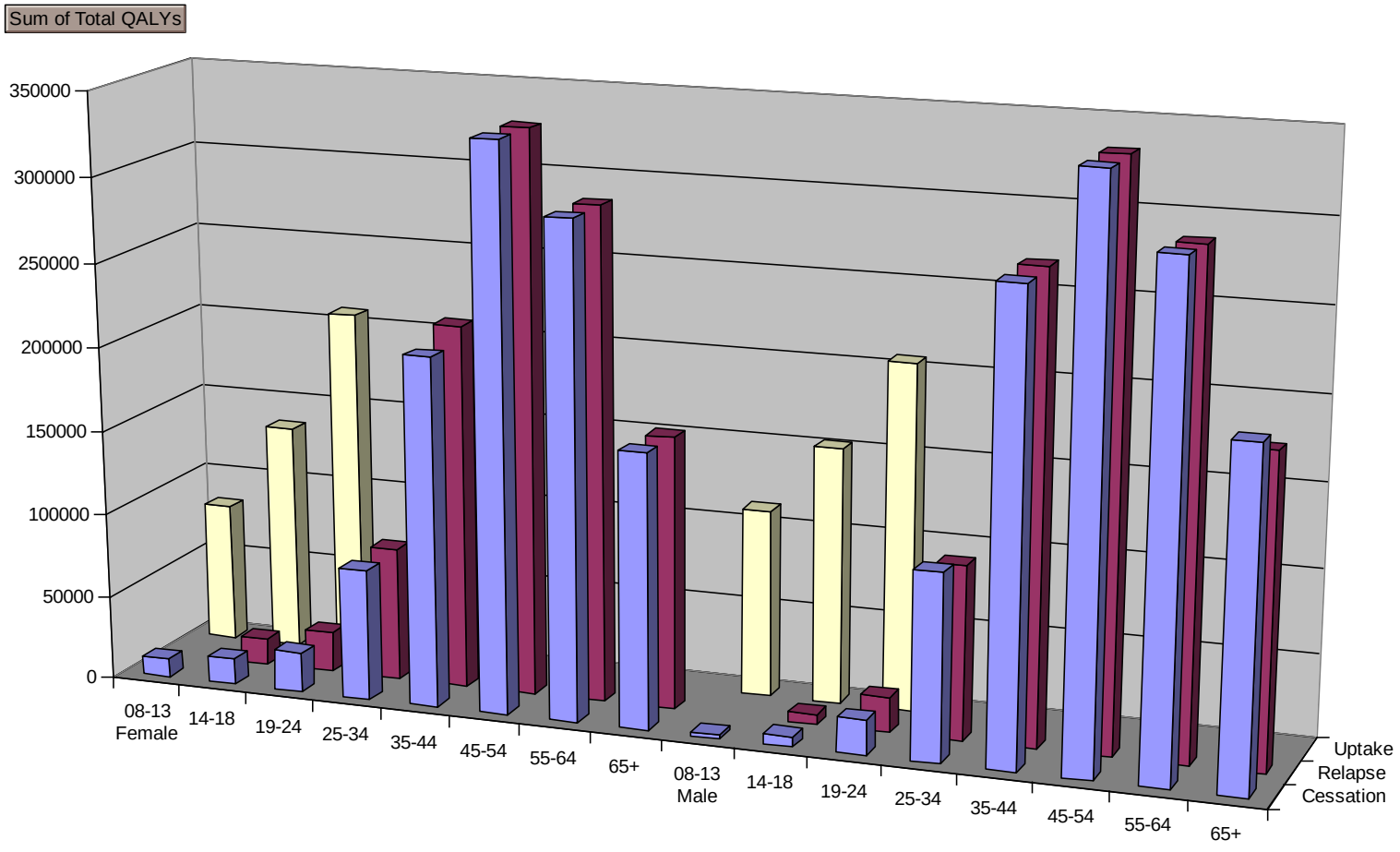
- Each axis represents a % change in a particular parameter
  - This proportional change is identical for the different parameters
- The distance assumed by the curve along that axis represents the magnitude of response to that change
  - Note that these sensitivities will depend on the state of system!

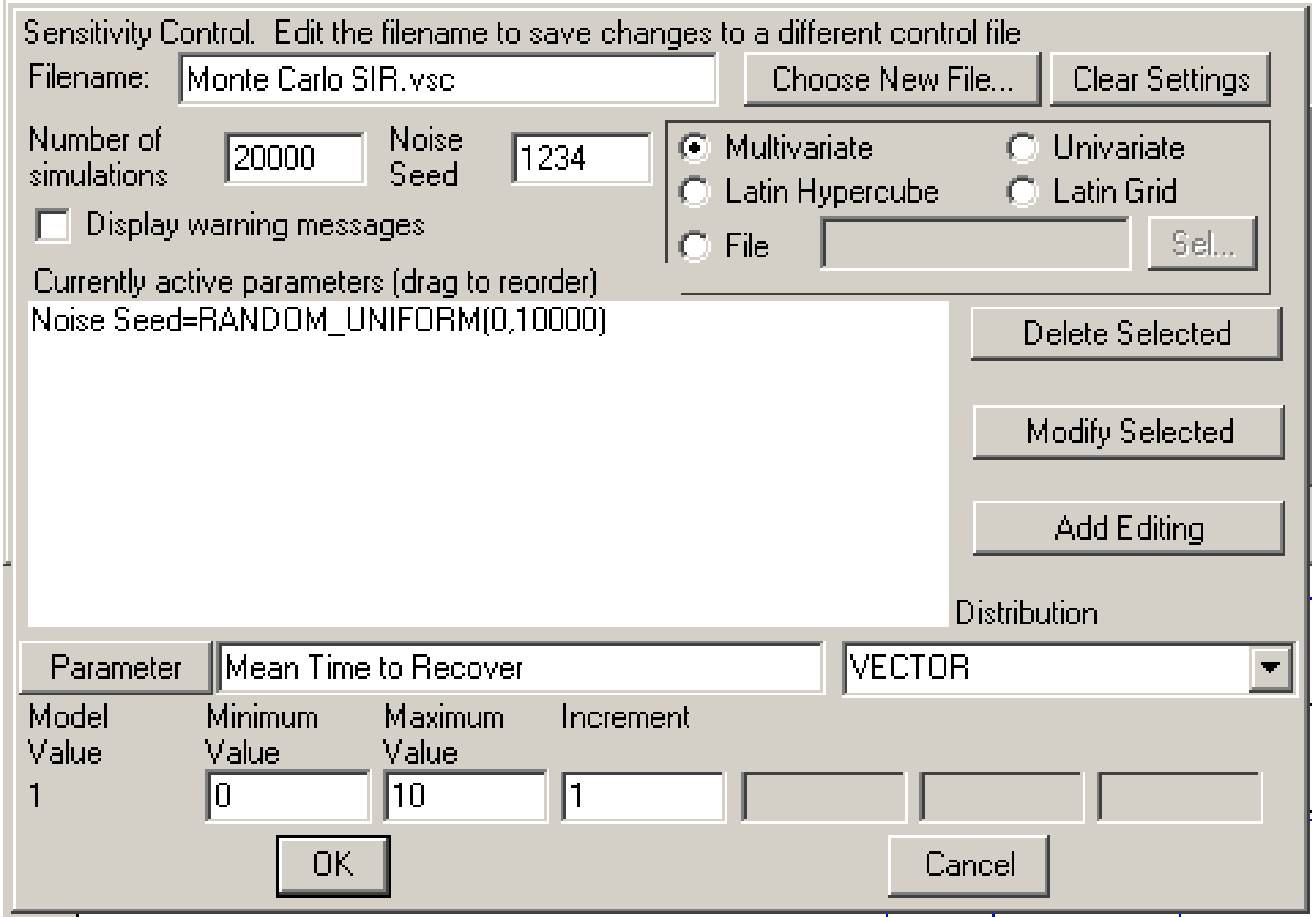
http://www.niwotridge.com/images/BLOGImages/SpiderDiagram.jpg

Systematic Examination of Policies



Tengs, Osgood, Lin

Sensitivity Analyses in Vensim



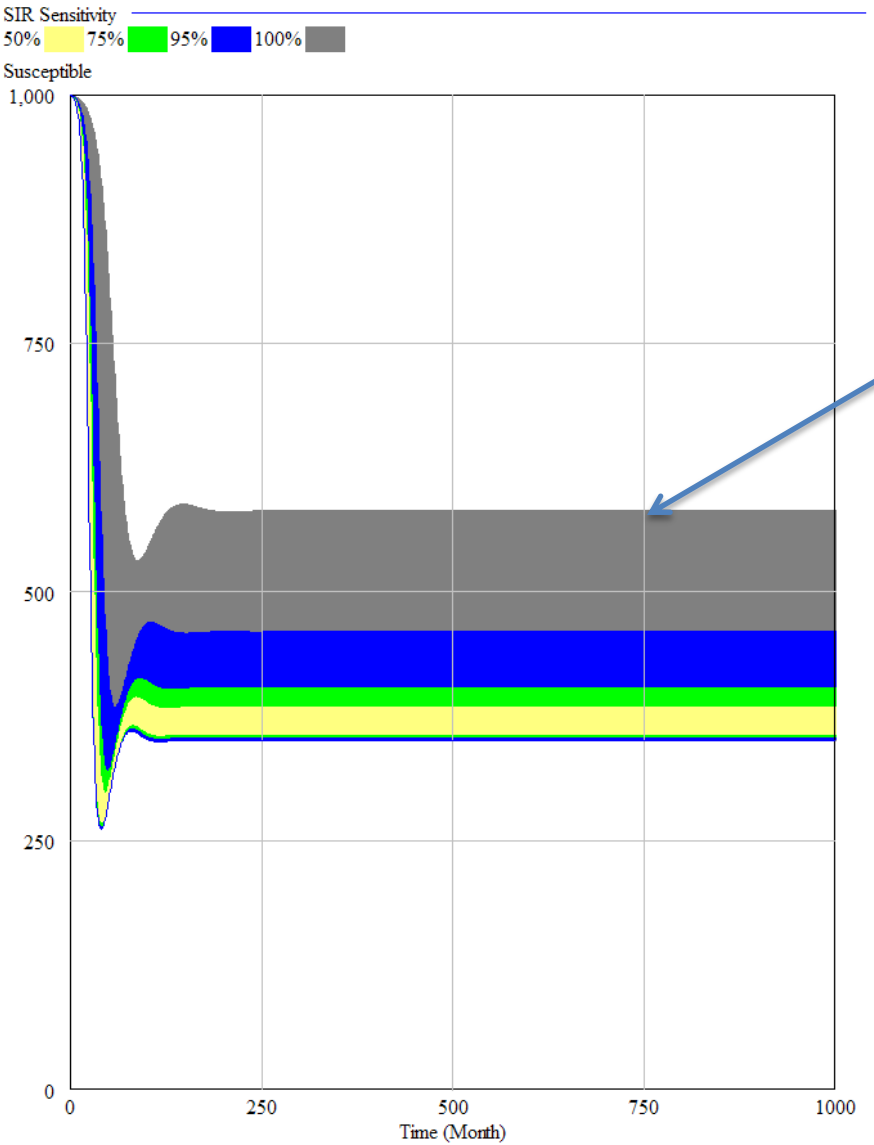
Sensitivity in Initial States

- Frequently we don't know the exact state of the system at a certain point in time
- A very useful type of sensitivity analysis is to vary the initial model state
- In Vensim, this can be accomplished by
  - Indicating a parameter name within the "initial value" area for a stock
  - Varying the parameter value
- In an agent-based model, state has far larger dimensionality
  - Can modify different numbers of people with characteristic, location of people with characteristic, etc.

Imposing a Probability Distribution Monte Carlo Analysis

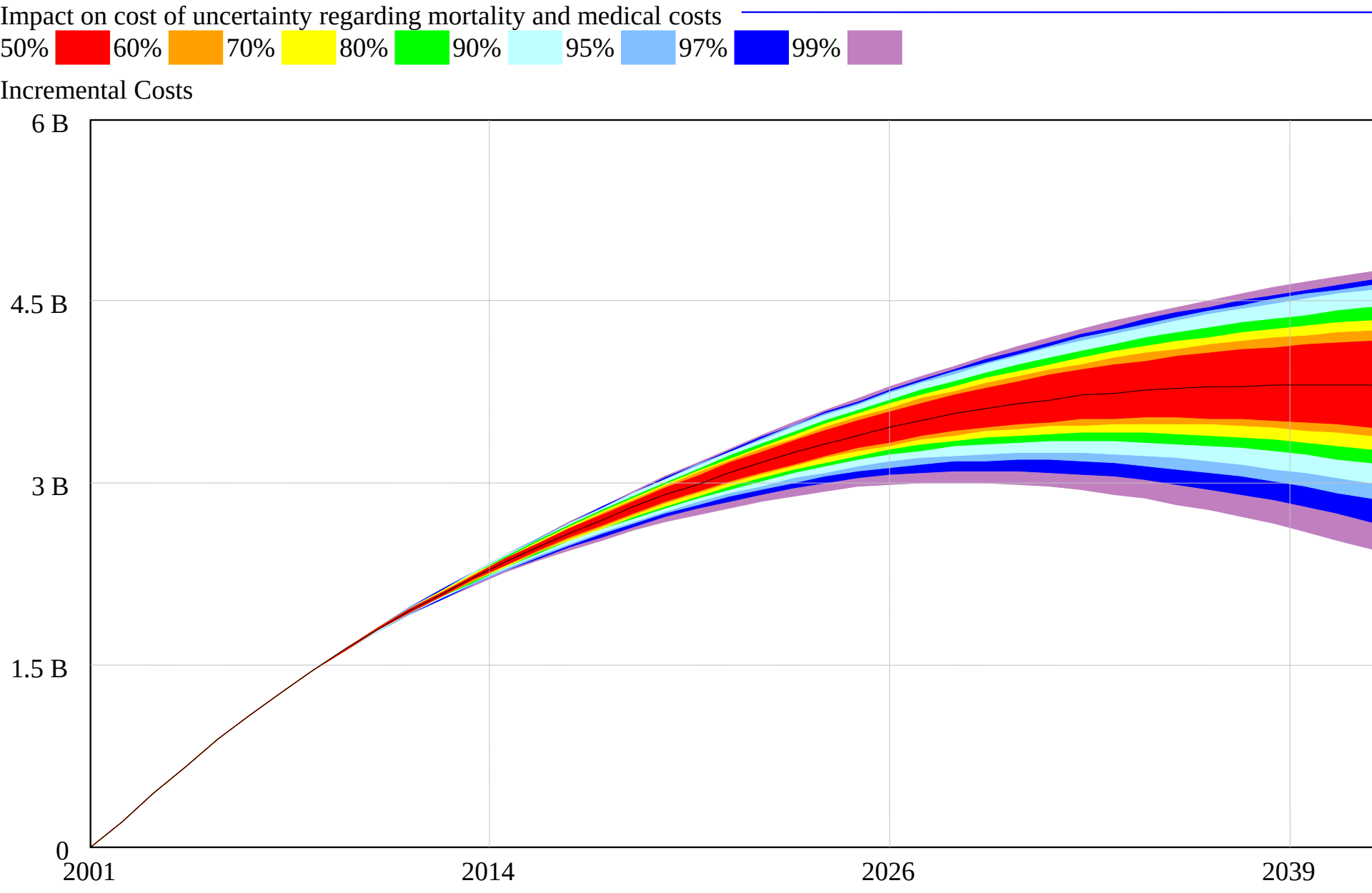
- We feed in probability distributions to reflect our uncertainty about one or more parameters
- The model is run many, many times (realizations)
  - For each realization, the model uses a different draw from those probability distribution
- What emerges is resulting probability distribution for model outputs

Example Resulting Distribution



Empirical Fractiles

Static Uncertainty



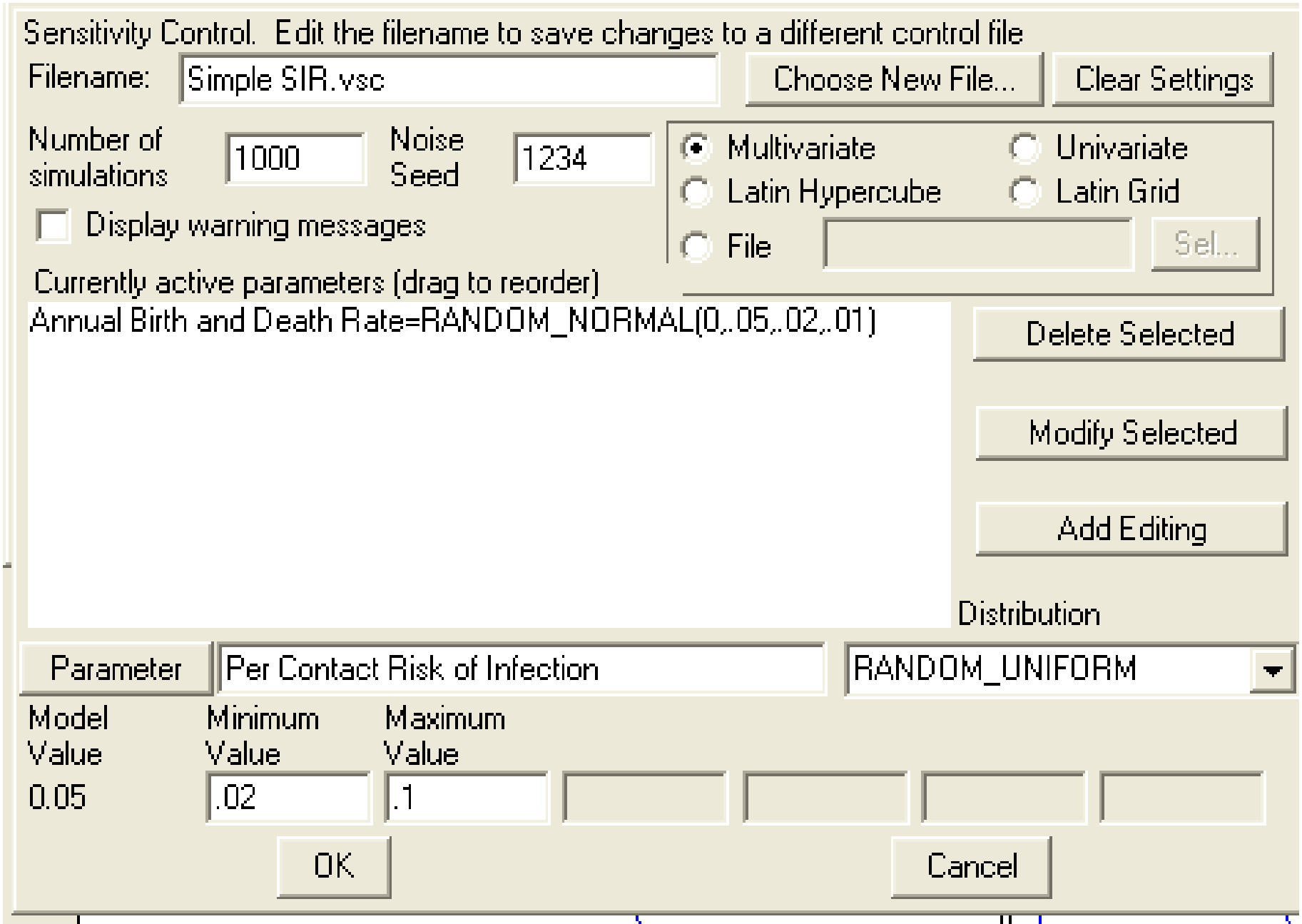
Multi-Way Sensitivity Analyses

- When examining the results of changing multiple variables, need to consider how multiple variables vary together
- If this covariation reflects dependence on some underlying factor, may be able to simulate uncertainty in underlying factor

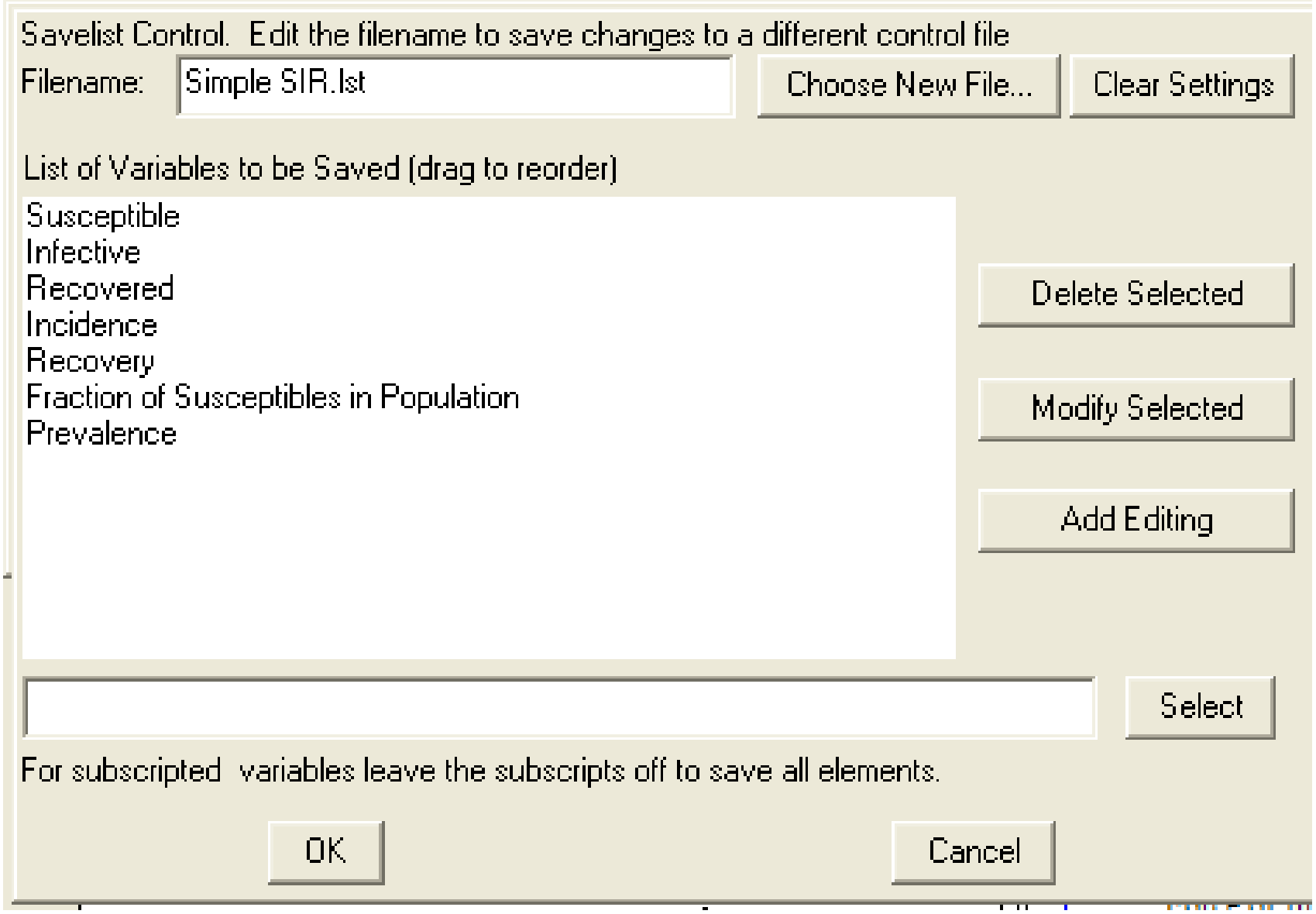
Performing Monte Carlo Sensitivity Analyses in Vensim

- Need to specify three things
  - The parameters to vary
  - How to vary those parameters
  - Which model variables to save away

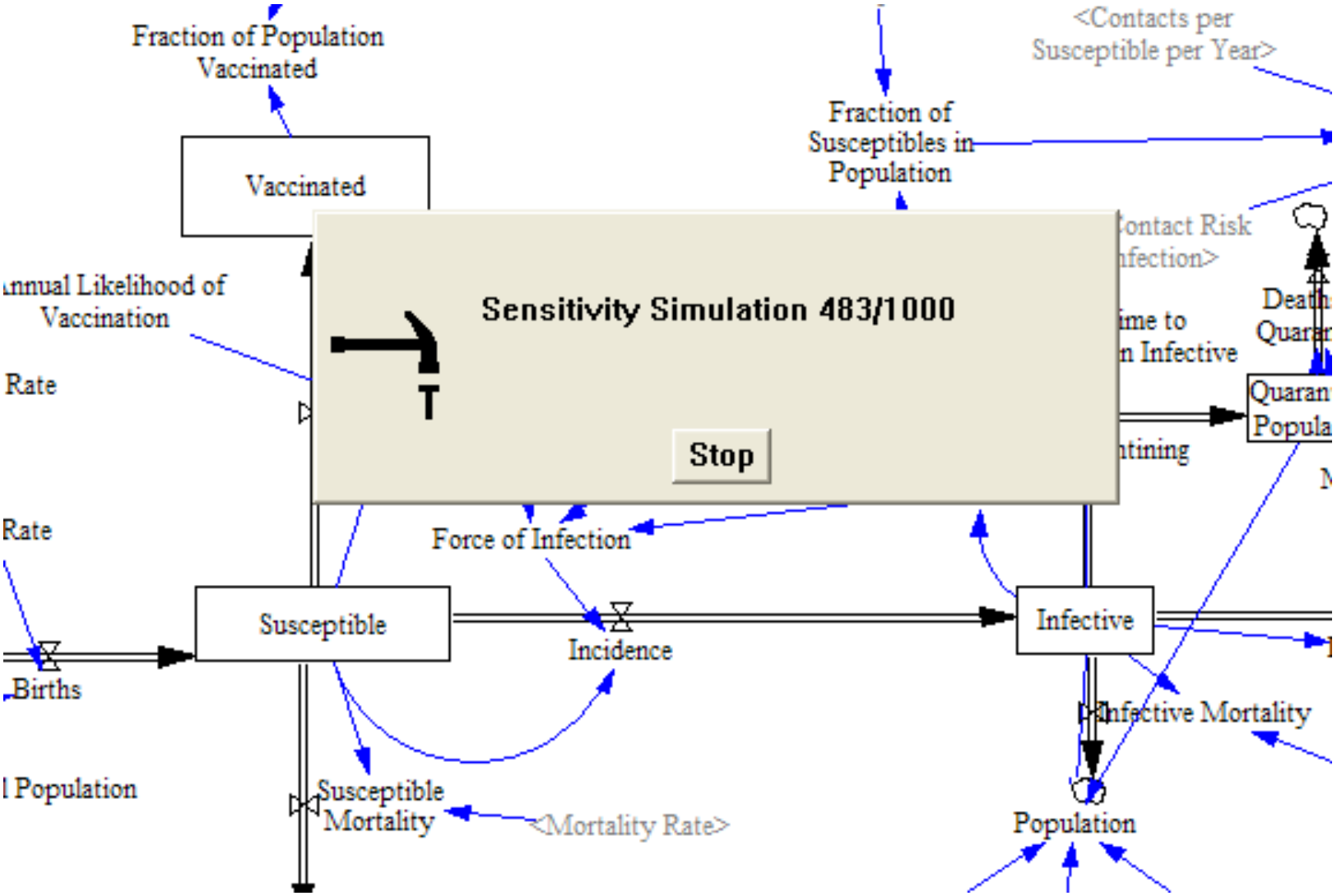
How & What Parameters to Vary



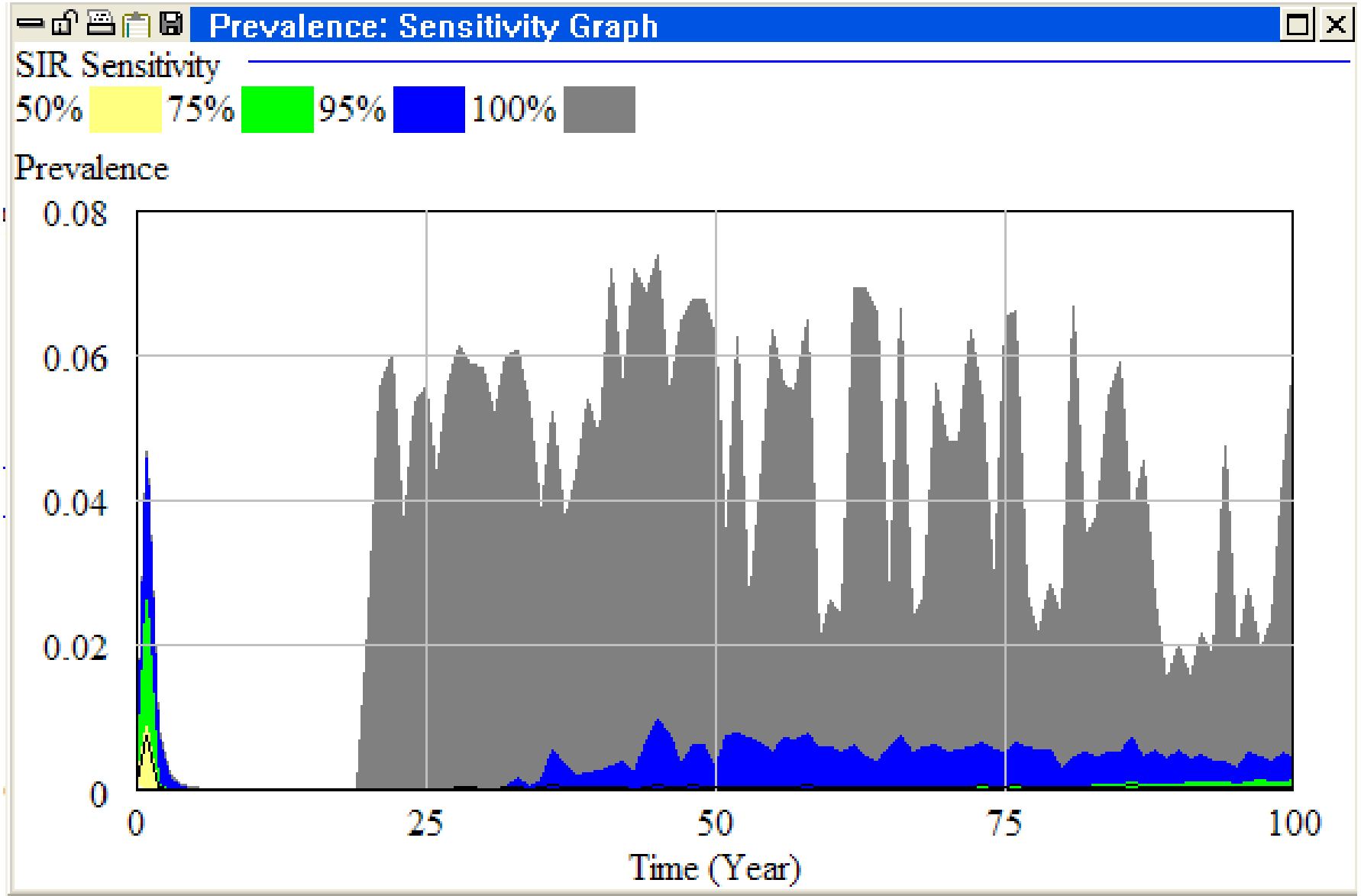
Model Values to Save Away



Monte Carlo Analyses

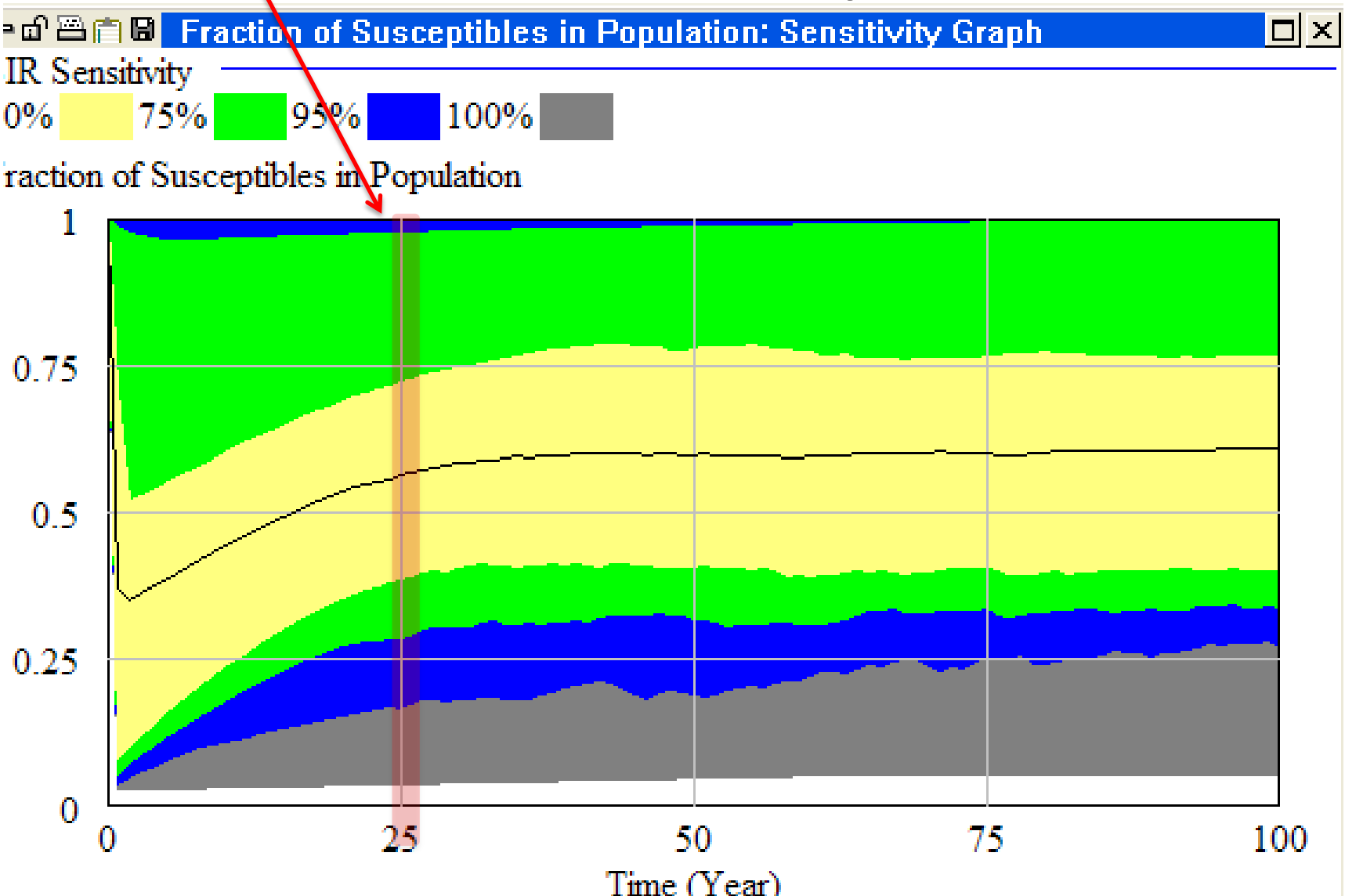


Sensitivity Results (Prevalence)



An observation at this point in time would produce a histogram (approximating a distribution) for fraction of susceptibles

Sensitivity Results (Fraction of Susceptibles)



Stochastic Processes

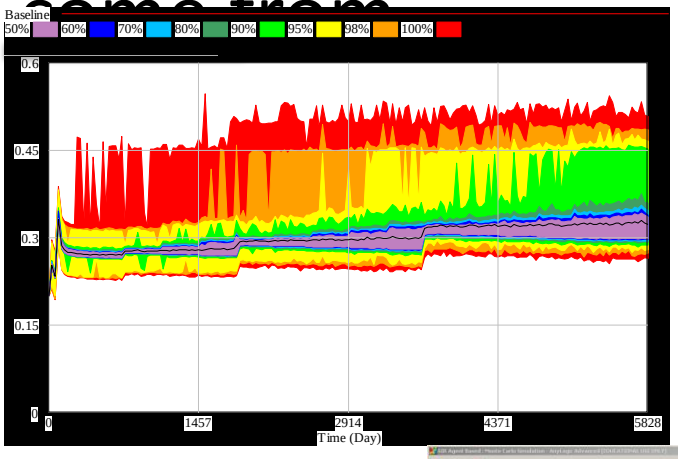
- System Dynamics models are traditionally determnistic
- As will be discussed in tutorials, models can be made stochastic
  - Transitions between states (e.g. duration of infection or immunity)
  - Transmission of infection
- As a result, there will be variation in the results from simulation to simulation

Summarizing Variability

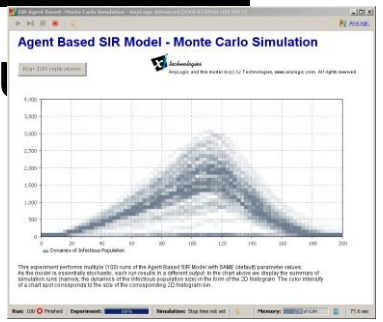
- To gain confidence in model results, typically need to run an ensemble of realizations
  - Deal with means, standard deviations, and empirical fractiles
  - As is seen here, there are typically still broad regularities between most runs (e.g. rise & fall)
- Need to reason over a population of realizations $\Rightarrow$ statistics are very valuable
  - Fractile within which historic value falls
  - Mean difference in results between interventions

Closing Question: How can we best adapt our policies to deal with ongoing uncertainty?

- We are dealing here with making decisions in an environment that changes over time
- This uncertainty could come from
  - Stochastic variability



  - Uncertainty regarding parameter valu



- There is an incredibly vast # of possible policies

Stochastic Processes in Vensim